Goos–Hänchen effect in the gaps of photonic crystals

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We show the presence of the Goos–Hänchen effect when a monochromatic beam illuminates a photonic crystal inside a photonic bandgap. © 2003 Optical Society of America

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When a light beam illuminates the interface between two homogeneous media under total internal reflection, the barycenter of the reflected beam does not coincide with that of the incident one: This is the Goos–Hänchen effect.[1] This phenomenon has been analyzed in its many guises, both theoretically[2–5] and experimentally.[1,6–8] In phenomenon's original form the incident beam comes from the medium with higher index in order to obtain total internal reflection. In this Letter we show that there is also a Goos–Hänchen shift when a monochromatic beam illuminates a photonic crystal, that is, a periodically structured device exhibiting photonic bandgaps.[9,10] Since the beams considered in nanophotonic devices are usually very narrow, this effect should be taken into account when designing such structures when the photonic bandgap phenomenon is involved. The Goos–Hänchen effect is linked to the variation of the phase of the reflection coefficient with the angle of incidence. In the case of total internal reflection the existence of evanescent waves explains the variations of the phase. Such an effect can be expected in photonic crystals in photonic bandgaps, where the Bloch waves behave much like evanescent ones. The main difficulty is finding the correct reflection coefficient.

We deal with one-dimensional (1D) (for instance, a stack of Bragg mirrors) or two-dimensional (2D) photonic crystals (for instance, a stack of diffraction gratings periodic in the x direction), which are finite in the y direction (located between the $y = 0$ and the $y = -h$ planes) and infinite in the x and z directions (see Fig. 1). We consider harmonic fields with a time dependence of $\exp(-i\omega t)$. We denote λ as the wavelength in vacuum and $k_0 = 2\pi/\lambda$ as the wave number in vacuum. If we consider only z invariant fields, the problem of diffraction is reduced to the study of the two usual polarized cases: $E_\parallel$ (electric field linearly polarized along z) and $H_\parallel$ (magnetic field linearly polarized along z).

The photonic crystal is illuminated by an incident Gaussian beam,[5]

$$u^i(x, y) = \int A(\alpha, W) \exp i\left(\alpha x + \sqrt{k_0{}^2 - \alpha^2}\, y\right) \mathrm{d}\alpha\,, \quad (1)$$

where

$$A(\alpha, W) = \frac{W}{2\sqrt{\pi}} \exp\left[-\frac{W^2}{4}(\alpha - \alpha_0)^2\right], \quad (2)$$

and $\alpha_0 = k_0 \sin\theta_0$, with θ_0 being the mean angle of incidence of the beam (Fig. 1).

For a 1D crystal there is only one reflected (and hence transmitted) order of diffraction. Therefore for an incident plane wave of wave vector $\mathbf{k} = k_0(\sin\theta, -\cos\theta)$ the field outside the crystal can be written

$$\begin{aligned} u(x, y) &= \exp[ik_0(x\sin\theta - y\cos\theta)] \\ &\quad + r(k_0, \theta)\exp[ik_0(x\sin\theta + y\cos\theta)]\,, \quad \text{for } y \geq 0 \\ u(x, y) &= t(k_0, \theta)\exp\{ik_0[x\sin\theta - (y + h)\cos\theta]\}, \quad \text{for } y \leq -h \end{aligned}$$

For a 2D crystal, when the period along the x axis is smaller than $\lambda/2$, there is only one reflected (and

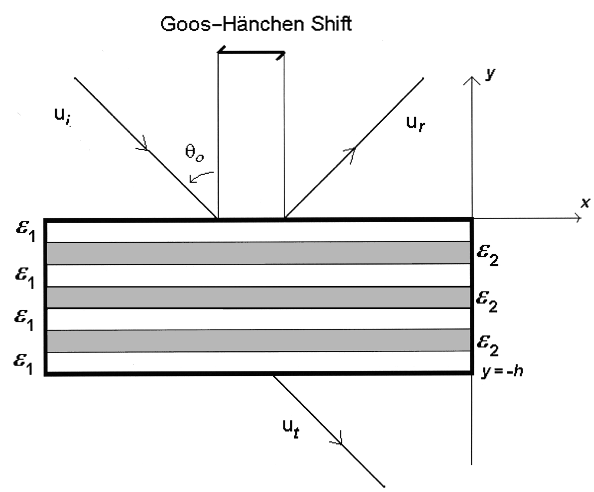

Fig. 1. One period of the photonic crystal consists in two layers of height d and of permittivity $\varepsilon_1 = 11.56$ and $\varepsilon_2 = 1$. The Goos–Hänchen shift is the distance between the centers of the incident and reflected beams.

hence one transmitted) propagating order of diffraction for each plane wave constituting the beam. Since there are evanescent waves, the above expressions for the field are no longer rigorous—they represent an approximation that holds far enough from the crystal.

It is then possible to characterize the electromagnetic properties of the structure by simply deriving its transfer matrix, with the considered structure being a 1D or a 2D crystal. More precisely,[11] there exists only one real matrix $\mathbf{T}(k_0, \theta)$ such that

$$\mathbf{T}\begin{bmatrix} 1 + r \\ i\beta_0(1 - r) \end{bmatrix} = t\begin{bmatrix} 1 \\ i\beta_0 \end{bmatrix}, \tag{3}$$

where $\beta_0 = k_0 \cos\theta$. This matrix gives an effective description of the medium as seen by the incident field.

The matrix $\mathbf{T}$ is real and has a determinant[11] that is equal to 1. The eigenvalues of $\mathbf{T}$ are thus the roots of the polynomial $X^2 - tr(\mathbf{T}) + 1$, which has real roots if $|tr(\mathbf{T})| > 2$. The product of these roots is equal to 1. One of the eigenvalues is smaller than 1 in modulus, and we will denote it μ. The other is equal to μ^{-1}.

For a 1D photonic crystal we denote T_0 as the transfer matrix for a period. For the whole structure containing N periods the transfer matrix T is equal to T_0^N. We denote κ as the eigenvalue of T_0 whose modulus is smaller than 1 ($\mu = \kappa^N$). Then the amplitude of the field is simply decreased by a factor κ each time it crosses a period of the structure. The field thus behaves like an evanescent wave without being one, strictly speaking.

Finally, for 1D and 2D structures $|tr(\mathbf{T})| > 2$ implies that (k_0, θ) is in a forbidden band. In this case we denote $\mathbf{v} = (v_1, v_2)$ [$\mathbf{w} = (w_1, w_2)$] as an eigenvector associated with μ (μ^{-1}). By solving Eq. (3), we obtain the following form for the reflection and transmission coefficients:

$$r(k_0, \theta) = \frac{(\mu - 1)f}{\mu^2 - g^{-1}f}, \qquad t(k_0, \theta) = \frac{\mu(1 - g^{-1}f)}{\mu^2 - g^{-1}f}, \tag{4}$$

where the functions f and g are defined by

$$g(k_0, \theta) = \frac{i\beta_0 v_1 - v_2}{i\beta_0 v_1 + v_2}, \qquad f(k_0, \theta) = \frac{i\beta_0 w_1 - w_2}{i\beta_0 w_1 + w_2}. \tag{5}$$

Since $\mu < 1$, then $\mu^{2g/f} < 1$ and $\frac{1}{1-\mu^2 g f^{-1}}$ can be considered an infinite sum. Thus the coefficients become[12]

$$r(k_0, \theta) = g + (g - f)\sum_{m=1}^{+\infty} \mu^{2m} g^m f^{-m}, \tag{6}$$

$$t(k_0\theta) = (1 - gf^{-1})\sum_{m=0}^{+\infty} \mu^{2m} g^m f^{-m}. \tag{7}$$

The physical meaning of these series is well known; they represent the multiple reflections[12] inside the photonic crystals on the $y = 0$ and $y = -h$ planes, leading to the fact that an infinite number of beams is transmitted and reflected (although, of course, with rapidly decreasing amplitude). Here we are only interested in the first reflected beam. The above result means that this beam behaves for a 1D crystal as if the structure were semi-infinite (since g is the reflection coefficient of the semi-infinite crystal). For a 2D crystal g is not exactly the reflection coefficient of a semi-infinite structure, although it tends toward this coefficient when $h \to +\infty$.

The beam can finally be written as

$$u^d(x, y) = \int A(k_0 \sin\theta, W) g(k_0, \theta) \times \exp[ik_0(x \sin\theta + y\cos\theta)]\cos\theta \mathrm{d}\theta. \tag{8}$$

Since $\mathbf{T}$ is a real matrix and μ is real as well, v_1 and v_2 are also real and $|g| = 1$. Therefore g can be written as

$$g(k_0, \theta) = \exp[i\phi(k_0, \theta)]. \tag{9}$$

The Goos–Hänchen shift is the distance between the centers of the incident and reflected beams. Since the center of the incident beam is located at $x = 0$, the shift can be written as

$$G_r = \frac{\int x|u^d(x, 0)|^2 \mathrm{d}x}{\int |u^d(x, 0)|^2 \mathrm{d}x}. \tag{10}$$

With the Parseval–Plancherel lemma we get

$$G_r = -\frac{\int A^2(\theta, k_0 W)\partial_\alpha \phi(\alpha)\cos\theta \mathrm{d}\theta}{\int A^2(\theta, k_0 W)\cos\theta \mathrm{d}\theta}. \tag{11}$$

Assuming a sufficiently large waist, we get

$$G_r \sim -(k_0 \cos\theta)^{-1}\frac{\partial\phi}{\partial\theta}. \tag{12}$$

This result is identical in form to that obtained for homogeneous media.

In fact, it can be shown that Eq. (6) is still valid for (k_0, θ) outside the gap, in which case g is still defined

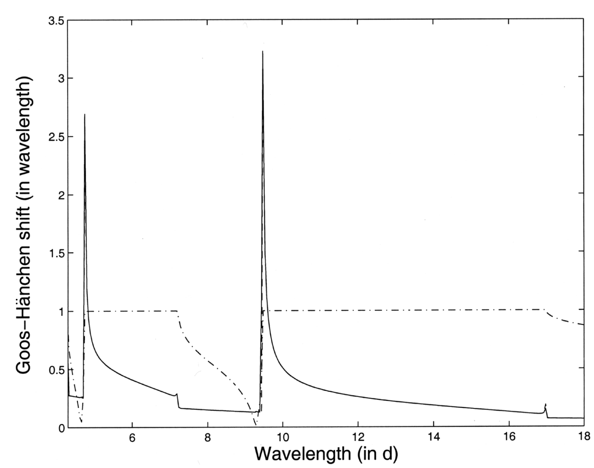

Fig. 2. Goos–Hänchen shift for a Gaussian beam of waist 10λ under a 50° angle of incidence for a wavelength $\lambda/d \in [4.5, 18]$ (the height of a layer being of size 1). The dashed-dotted curve represents $|g|$ so that the gaps, characterized by $|g| = 1$, can be easily identified by the reader. The dashed curve represents the derivative of the phase of g, which can barely be distinguished from the shift for a Gaussian beam (solid curve).

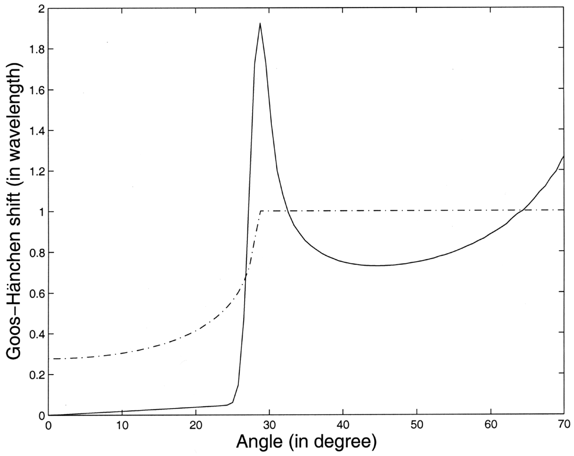

Fig. 3. Goos–Hänchen shift (normalized by $\lambda = 10$) for a Gaussian beam for an angle of incidence $\theta \in [0, 70]$. The dashed-dotted curve represents $|g|$.

as in Eq. (5) but is no longer of modulus 1. In this case $\overline{g} = f$, and g is chosen such that $|g| < 1$. Moreover, g is a continuous function of (k_0, θ).

We computed the Goos–Hänchen shift for a 1D photonic crystal illuminated by a Gaussian beam with a 50° angle of incidence. The crystal is presented in Fig. 1, and the shift versus the wavelength is shown in Fig. 2. As expected, the shift is important in the gaps. More precisely, it presents a peak at the left side of each gap caused by a swift variation of the phase of g. This phenomenon has much in common with what happens in the case of total internal reflection. The phase of the reflection coefficient indeed presents such a behavior near the limit angle. Let us consider the Goos–Hänchen shift when $\lambda/d = 10$ is fixed and when the angle of incidence may vary. It can be seen in Fig. 3 that small angles correspond to couples (k_0, θ) outside the gap. But when the angle of incidence increases, the structure enters the gap.[13] At the edge of the gap the phase is subject to rapid variations, leading to a large shift of the outgoing beam, as in the case of total internal reflection.[5]

In conclusion, we have theoretically demonstrated the presence of the Goos–Hänchen effect in the gaps of photonic crystals and provided theoretical tools to deal with such an effect. We have exhibited the function defined by Eq. (5), which is the correct reflection coefficient to be considered. Our numerical computations for a one-dimensional photonic crystal show that the shift can indeed be found for values of λ and θ in a gap. The shift is important when either λ or θ are varied to cause the structure to enter the gap. In the latter case the phenomenon has much in common with the total internal reflection near the limit angle. This effect could play an important role in structures such as that described in Ref. 14, where precise knowledge of the trajectories of the reflected or refracted beams is needed for the structure to work properly.

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