

Lens equation for flat lenses made with hyperbolic metamaterials

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This study aims to give a general theory that enables the design of flat lenses based on hyperbolic metamaterials. We derive a lens equation that is demonstrated to involve the curvature of the dispersion relation. Guided by this theory, hyperbolic lenses of focal length ranging from zero to a few wavelength are simulated. High transmission efficiency is also obtained by reducing the amount of metal compared to the dielectric material. © 2012 Optical Society of America

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Metamaterials play a crucial role in recent development of advanced photonic systems. Their success stems from their incomparable faculty to tailor the optical effective response of matter [1]. This feature, combined with transformation optics techniques, has allowed for the realization of cloaking devices or novel lenses [2,3]. In the latter case, Pendry's pioneer work has shown that flat slabs with a negative refractive index allow for the focalization of subwavelength images [4]. This concept has been realized for frequencies ranging from THz to GHz in metamaterial-based devices that present either a negative permittivity or both negative permittivity and permeability [5,6]. Such a flat lens with negative effective index mimics a homogeneous medium with a parabolic dispersion relation $k_x^2 + k_y^2 = \epsilon\mu\omega^2/c^2$. In that case, the lens equation is simply related to the thickness e of the flat lens, and the object and image focal lengths, f_o and f_i , are given by $f_o + f_i = e$ [7]. Hyperbolic metamaterials have then quite naturally emerged as an alternative approach to yield subwavelength images. These anisotropic materials, which can be achieved by stacking dielectric and metallic layers, actually present a hyperbolic dispersion relation $k_x^2/\epsilon_y + k_y^2/\epsilon_x = \omega^2/c^2$ with $\epsilon_y < 0$ and $\epsilon_x > 0$. Near-field [8] or far-field [9] subwavelength images can be formed using hyperbolic metamaterials. In the case of near-field focalization [8], a canalization mechanism has been proven to transport both propagating and evanescent waves emitted by the source in a self-collimation regime. An image is formed right at the end of the flat lens with a theoretical resolution of $\lambda/60$. This super resolution is attributed to the hyperbolic dispersion $k_y = \sqrt{\epsilon_x} \sqrt{(\omega^2/c^2) + (k_x^2 + |\epsilon_y|)}$ that enables the propulsion of propagating waves of real wavevector k_y whatever the value of the transverse wavevector k_x . Hyperbolic lenses convert the evanescent waves radiated by the object into propagating waves carrying subwavelength details. The conjugate points for such a lens are however located right on the interfaces of the structure, which leads to place both object and image at the vicinity of the hyperbolic lens interfaces. This is also required to obtain subwavelength resolved images since a part of the transmitted spectrum is converted back into evanescent waves at the end interface. Because of this conversion

mechanism, subwavelength resolution is only possible at near-field distance smaller than $\lambda/4$. This process differs from the evanescent amplification in perfect lenses that theoretically allows to focalize an image at distances larger than λ with a super resolution. However, in both parabolic and hyperbolic lenses the presence of plasmonic modes have been identified to strongly contribute to the resolution [10]. Beyond their near-field properties, hyperbolic metamaterials have also been used to make hyperlenses that provide far-field images with subwavelength resolution [9,11–13]. As depicted in several works, this property derives from the compression of the angular frequency, which allows in return to magnify the object [14,15]. Hyperbolic metamaterials would seem to only operate in those two extremes regimes (near- or far-field) if Scalora and coauthors had not demonstrated that the focalization at distances of $\lambda/2$ was possible [16]. However, to date, there is no theory that can help in the design of flat hyperbolic lens whose focal length is sized at will. To address this issue, we apply a beam propagation theory recently developed to demonstrate resonant and slow light self-collimation in layered media [17,18]. This semiclassical theory, successfully applied for hyperlenses [19], allows us to derive the lens equation of hyperbolic lenses and unifies previous results [8,16]. Our approach even leads to an overall improvement of the transmission efficiency of such lenses despite the optical losses induced by the metallic layers.

Let us start by setting the optical conditions required to design a hyperbolic lens. As discussed previously, a hyperbolic dispersion characterized by an anisotropic permittivity tensor of diagonal elements $\epsilon_x > 0$ and $\epsilon_y < 0$ is required. In the homogenization regime and for TM polarization (out of plane magnetic field), Maxwell-Garnet's formulas give these components in terms of thicknesses d_1, d_2 and permittivities ϵ_1, ϵ_2 of the dielectric and metallic slabs:

$$\epsilon_x = \frac{\epsilon_1 d_1 + \epsilon_2 d_2}{D}, \quad \epsilon_y = \left[\frac{\epsilon_1^{-1} d_1 + \epsilon_2^{-1} d_2}{D} \right]^{-1}, \quad (1)$$

where $D = d_1 + d_2$ is the lattice period. At optical frequencies, the negative permittivity ϵ_2 of metals allows the design of a hyperbolic medium of appropriate permittivity

components when the following conditions, derived from Eq. (1), are satisfied:

$$\frac{|\epsilon_2|}{\epsilon_1} < \frac{d_2}{d_1} < \frac{\epsilon_1}{|\epsilon_2|}, \quad -\epsilon_1 < \epsilon_2 < \epsilon_1. \quad (2)$$

Here, we chose diamond and silver layers with permittivities equal to $\epsilon_1 = 5.08$ for diamond and $\epsilon_2 = -1.78 + 0.6i$ for silver at 350 nm [20]. As shown by conditions (2), this permittivity contrast enables us to drastically decrease the amount of metal, reducing in turn the optical absorption of the lens. In the following, we will consider a lens consisting of $N = 20$ periods of hyperbolic metamaterials with $d_1 = 2.8d_2$, presenting a particularly good transmission efficiency. Figure 1 shows the transmission diagram computed for the fixed wavelength $\lambda = 350$ nm and for a varying period D . The transmission larger than 20% is enhanced by adding an antireflecting coating made of diamond layers of thickness $\lambda/(10\sqrt{\epsilon_1})$ placed at the boundaries of the structure.

Let us derive the lens equation and show that the focal length can be tuned from zero to a few wavelengths. The propagation of a Gaussian beam on a distance y in a medium with a dispersion relation $k_y(\omega, k_x)$ can be written

$$U(x, y) = \int_{-\infty}^{\infty} dk_x \tilde{U}(k_x, 0) e^{ik_x x} e^{ik_y(k_x) y}, \quad (3)$$

where $\tilde{U}(k_x, 0) = W_0/(2\sqrt{\pi}) \exp(-(k_x W_0/2)^2)$ is the angular spectrum of the Gaussian beam. The dispersion relation can be approximated for small values of k_x by

$$k_y(\omega, k_x) = k_y(\omega, 0) - \frac{k_x^2}{2k_0 n_c}, \quad (4)$$

where the curvature index [17,18] of the medium is given by $n_c = -k_0(\partial^2 k_y / \partial k_x^2|_{k_x=0})^{-1}$ with $k_0 = 2\pi/\lambda$. For a homogeneous medium, n_c reduces to the refractive index. Equation (3) can be used, along with the quadratic approximation for the dispersion relation, to describe the propagation of a Gaussian beam on a distance f_0 in air between the object focus and the lens, inside the multilayer considered as a homogeneous medium, and finally on a distance f_i in air between the lens and the image (Fig. 1). This finally yields

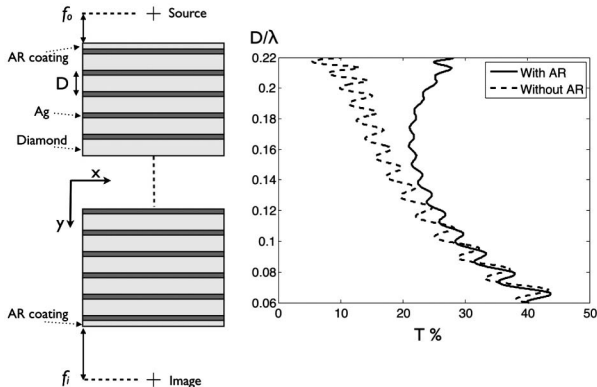


Fig. 1. Design and transmission diagram of the hyperlens with and without antireflection coating (AR).

$$U(x, L) = e^{i(k_y(\omega, 0))L} \int_{-\infty}^{\infty} dk_x U(k_x, 0) e^{ik_x x} e^{-iL \frac{k_x^2}{2k_0} \left\langle \frac{1}{n_c} \right\rangle}, \quad (5)$$

where $L = f_o + ND + f_i$ is the total distance between the object and the image, $\langle k_y(\omega, 0) \rangle L$ represents the mean phase, and the average curvature index is given by $\langle 1/n_c \rangle = L^{-1}(f_o + ND/n_c + f_i)$. When the average curvature vanishes, $\langle 1/n_c \rangle = 0$, Eq. (5) simplifies to $U(x, L) = U(x, 0) e^{i(k_y(\omega, 0))L}$. This demonstrates that the beam is phase delayed and retrieves its initial waist W_0 when the following lens equation is satisfied:

$$f_o + f_i = -\frac{ND}{n_c}. \quad (6)$$

The focal lengths are then directly determined by the curvature index of hyperbolic dispersion relation. In the homogenization regime, the curvature index is driven by the effective permittivities: $n_c = \epsilon_y / \sqrt{\epsilon_x}$. The canalization regime proposed by Belov stands in flat iso-frequency curves (IFCs) obtained when $\epsilon_y \rightarrow -\infty$. In that case as shown by Eq. (6), the focal distance tends to zero so that the object and the image are located at the vicinity of the lens interfaces. However, it has been also shown [16] that such a hyperbolic lens can also focalize light at a distance of $\lambda/2$. This result can be explained by considering the curvature index beyond the long wavelength regime. For that purpose, the IFCs are computed for our structure by taking into account the metal losses in the dispersion relation of 1D photonic crystals [17]. Figure 2(b) shows that the curvature index is positive in the long wavelength limit when metallic losses are taken into account. The canalization regime is obviously perturbed by the introduction of optical losses since the flatness of the IFC at $D/\lambda = 0.05$ is lost around $k_x = 0$, Fig. 2(a). The central propagating spectrum of the beam diverges, showing the negative impact of losses on the resolution. At the reduced frequency $D/\lambda = 0.1$ the curvature index diverges to infinity, since a flat IFC is met. Beyond this frequency, the curvature index ranges from large negative values to zero when reaching a photonic band gap between the first and second bands. This wide variation of n_c , which cannot be anticipated with the homogenization approach, provides us a new way to design hyperbolic lenses of focal distance scaled between zero and a few wavelengths. To demonstrate this property, the theoretical image-focal distance f_i computed from Eq. (6) is compared with the simulated one (Fig. 3).

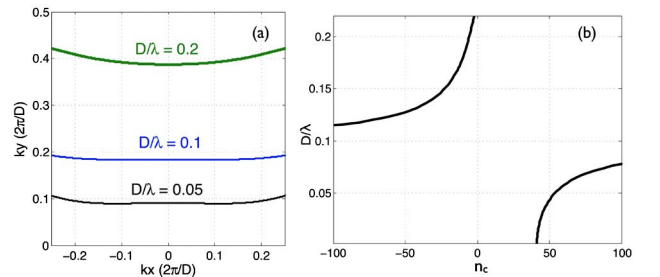


Fig. 2. (Color online) (a) IFCs computed for Ag/Diamond layers with the metal losses. (b) Curvature index versus the reduced frequency.

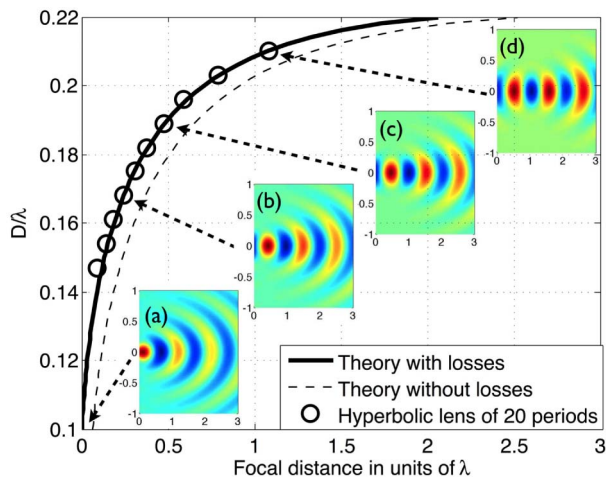


Fig. 3. (Color online) Focal distance versus de reduced frequency D/λ . The bold and dashed lines obtained with Eq. (6) are computed with and without silver losses. The circles corresponds to the focal distance f_i computed for a hyperbolic lens of 20 periods. (a)–(d) Magnetic field maps of axis in units of λ . The image is focalized at the focal distances $0, \lambda/4, \lambda/2$ and λ .

The propagation of a Gaussian beam of waist $W_0 = \lambda/5$ focalized at the distance $f_o = 0.1D$ from the lens is computed with a modal method, which enables us to extract the image-focal distance f_i [21]. Both results are in agreement and show that our hyperbolic lens can make images focused from the lens interface to one wavelength. The silver losses are seen to slightly decrease the focal distance of the structure. The magnetic field maps attest to the quality of the images focalized at distances $f_i = \{0, \lambda/4, \lambda/2, \lambda\}$ and obtained for hyperbolic lenses of periods $D = \{35, 59, 66, 70\}$ nm. In the last case, despite the large thickness of the lens (1400 nm), the total amount of silver reduced here to 369 nm, which explains the high transmission efficiency larger than 25% (Fig. 1). The resolution of those hyperbolic lenses is evaluated by computing the relative variation of the waist for each focal distances. We have found that $\Delta W/W_0$ varies from 0.4 to 1.8 when the focal distance increases from zero to one wavelength. Subwavelength resolution (a waist smaller than $\lambda/2$) is obtained as long as the focal length is smaller than 0.7λ , which is a good result regarding the optical losses.

In conclusion, we derived the lens equation of hyperbolic lenses and showed that the focal length can be

adjusted on-demand. Beyond the homogenization regime, light focalization is demonstrated to be driven by the local dispersion curvature. This theory allows the transmission efficiency to be optimized by a drastic reduction of the metal amount that decreases optical losses.

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